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The self-attraction effect in absolute gravimeters and its influence on the CIPM key comparisons during the ICAG2009

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Abstract[1]

This report discusses the following issues:

1) the self-attraction effect (SAE) in absolute gravimeters
2) the influence of the SAE on the evaluation of the ICAG-2009 results
3) the correction for the SAE to be applied to the ICAG-2009 results presented in the CCM.G-K1 Draft A report

[1] This Report BIPM-2012-01.doc is based on TM193V5b/ 21/2/2012 with English correcxtions
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I. Introduction

The removal of biases due to a variety of causes constitutes a major challenge in absolute measurements of gravity. The Technical Protocol (TP) [1] of the 2009 International Comparison of Absolute Gravimeters (ICAG-2009) considered these biases in detail and specified a set of conventional uncertainties based on the knowledge available in 2009 (see Annex D of TP). The standard uncertainty associated with the self-attraction effect (SAE) of absolute gravimeters was specified as 0.2 μGal[2], probably based on the FG5 estimate of 1995 [2], but the size of the correction was not given. The last three ICAGs (which took place in 2001 [3], 2005 [4] and 2009 [1]) have all used the uncertainty (0.2 μGal) but considered SAE=0.

Unfortunately, a study carried out by Robertson in 1996 [5] has not drawn enough attention by the participants of ICAGs. Since then there was no new investigations about the SAE of the most popular and commercialized AG, the FG5, until recently. A recent study [4] shows that the so-called SAE is about an order of magnitude bigger than its uncertainty as given in the TP, and may be 1.5 μGal to 2 μGal for the FG5. Although the SAE of some absolute gravimeters (AGs) has been estimated by their owners for their particular equipment, e.g. IMGC/INRIM, these participants however either withdrew from or did not participate in the key comparison of ICAG-2009. The SAE of other types of AG may be less but will not be negligible. Thus if the results obtained are not corrected for the SAE, they will be significantly biased by it.

In the last three ICAGs, measurements made using FG5 gravimeters have dominated the key comparison reference value (KCRV), representing 80 % to 90 % of the total weight. The combined Type A standard uncertainty of the KCRV for the CCM.G-K1 part of ICAG-2009 was estimated to be between 1.0 μGal and 1.3 μGal [3,4,7], which means that the bias due to the SAE of the FG5 instruments is bigger than the uncertainty of the KCRV. Consequently, the offsets of the AGs from the KCRV are more or less affected.

The present report assesses the size of the SAE and its influence on the ICAG2009 results. The result of this study will support its final evaluation and report.

II. Estimations of the self-attraction effect of absolute gravimeters

2.1 Background

Because absolute gravimeters are not massless, and indeed instrument components above and below the sensor can have quite large densities, they themselves exert a gravitational attraction on the sensor; the vertical component of the attraction is the so-called self-attraction effect. The corresponding correction to the gravity measurement to remove the SAE is the self-attraction correction (SAC).

Absolute gravimeters cannot operate without these components, and there is no physical method to test for self-attraction effects except through complex comparisons of different types of instruments. In general, an instrument will not be able to occupy the same space as another gravimeter without modifying the instrumental environment. The SAE of each gravimeter must therefore be estimated by modeling techniques. The uncertainty of such estimations depends on the model used.

Niebauer et al. [2] published an error (uncertainty in the modeling technique) of 0.1 μGal without providing an estimate of the actual self attraction of the new FG5 absolute gravimeter. Robertson estimated the self-attraction effect for the FG5 design to be +1.35 μGal and also found an uncertainty of 0.1 μGal. In his modeling, the author used hollow cylinders for all components, and he assumed homogeneous masses for each segment of the instrument. However the study of Robertson has not been taken into account in the earlier ICAGS.

2.2 Method

In geophysics, gravity modeling has provided users with tools to approximate the effects of ore-bearing bodies. Unfortunately most of these techniques, including the one used by Robertson, are not capable of accurately simulating “within” the mass itself, thus forcing the use of approximations.

With appropriate mathematical tools, the use of right rectangular prisms [6] can more closely simulate the effects of gravitational attraction even within the model itself without resorting to shortcuts. The same methodology has been applied here to simulate the self-attraction effect of all the absolute gravimeters participating in the ICAG-2009 and the FGC-1 used in ICAG-2005.

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[2] 1 Gal = 1 cm s^{-2}. The default unit used in this report is μGal

Narrow or tall right rectangular prisms can be modeled in place of cylindrical bodies such as vacuum chambers and "super-springs" because of symmetry in both body types. The sensors are usually at the exact center of these instruments, thus reducing the approximation errors. Even so, this approximation will be part of the uncertainty budget of this technique.

A special case was made for the FG5s as these gravimeters have evolved significantly since 1993. In particular, the interferometer component of the gravimeter has been reduced in size and weight, thus decreasing somewhat the self-attraction in the newer models. The vacuum chamber has also changed slightly, especially the steel component in the lower part of the chamber.

2.3 Numerical estimations

In [10] the SAE is carefully estimated for three common configurations of the FG5 gravimeter: two relate to the new FG5 (with fiber laser optics) depending on the dropper tripod used (with straight or sloping legs) and one to the old FG5 with bulk interferometer (for dropper tripod with sloping legs). The gravimeters were divided into a few principal parts for which the masses were measured. The total mass of each part was subdivided into individual components of homogeneous density in accordance with their dimensions and material. The shape of all gravimeter components was approximated by hollow right circular cylinders, right rectangular or triangular prisms all with defined wall thickness. The SAE of each individual component ($E_{\mathrm{SAE}i}$) can be obtained by integrating the vertical component of gravitational attraction over the volume V

$$E_{\mathrm{SAE}i} = G\,\rho_i \int_V \frac{\Delta z\, dv}{r^3} \quad ,$$

where G is the gravitational constant, r the distance between the volume element and the integration point , Δz the vertical distance from the integration point, and ρ_i the density of component i computed from its mass and volume.

The first and second authors of this report carried out independently the estimations 1 and 2 reported below for the main commercially available AGs. Estimation 1 also considered other types of absolute gravimeters, including the FGC-1, which participated in ICAG-2005 but not ICAG-2009. Some participants in the ICAG 2009 also estimated the SAE for their prototype AGs, their results are collated in section 2.3.3.

2.3.1 Estimation 1

The model was applied to seven models of absolute gravimeter: the FG5-106 instrument (with bulk interferometer) was used as an example of the old type of FG5, and FG5-236 (with fiber laser optics) as example of the new. The FG5s and the JILAg instruments have a "drop" range of 0.20 m, while that of the A-10 is 0.10 m and that of the FGC-1 is 0.03 m. The estimated SAEs are shown as a function of height in Figure 2.3.1 below.

In Table 2.3.1, the self-attraction effects for five types of instruments are listed at the top and bottom of their drop ranges. The uncertainties of the estimates range from 0.03 µGal to 0.27 µGal. These numbers were obtained by testing the effect of modifying the densities of different components for the FG5s and for JILAg, and by modifying the component positions for the A-10 and the FGC-1. These last two devices are more complex in design and the components are much closer to the sensing area than the FG5s and for JILAg instruments.

If any of these instruments were to be modified, as for example the new FG5X, then the calculations would have to be repeated.

Table 2.3.1 The self-attraction effect, E_{SAE}, of seven types of AG at the top and bottom of their drop ranges (the old or new types of FG5 are equipped with bulk interferometer or fiber laser optics, respectively)

Type of AG	Instr. Height /m	E_{SAE} /µGal	u_{SAE} /µGal	Note
FG5 (old)	1.29	1.78	±0.03	before around 1998
	1.09	1.67	±0.03	
FG5(new)	1.29	1.48	±0.03	since around 1998
	1.09	1.14	±0.03	
A-10	0.70	0.58	±0.27	
	0.60	1.55	±0.27	
JILAg	0.91	0.69	±0.08	
	0.72	1.23	±0.08	
FGC-1	0.235	-1.93	±0.06	
	0.215	-1.71	±0.06	
MPG-2	1.142	1.4	±0.20	
FGL	0.814	1.1	±0.11	

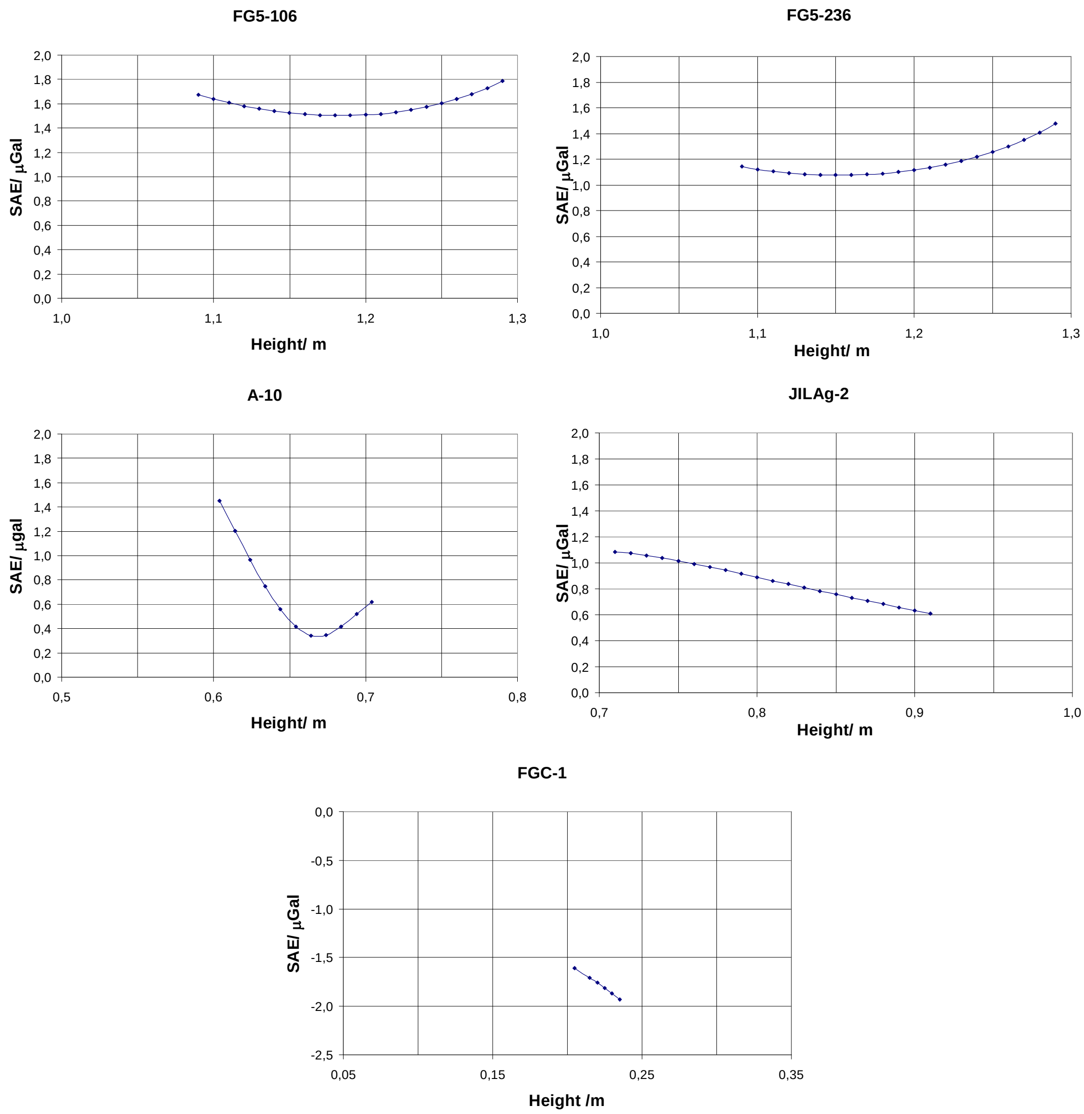


Figure 2.3.1. The SAE of the different types of AG as a function of height over the whole drop range

2.3.2 Estimation 2

In each case the integration point was defined as the reference height of the gravimeter (distance of the effective position of the free-fall above ground) [10]. The reference heights of the FG5s are on average 1.21 m or 1.22 m depending on the type of the dropper tripod (see Table 2.3.2). The uncertainty of the SAE is estimated to be 0.2 µGal, caused mainly by neglecting a few components in the dropping chamber and by approximation of the mass distribution within the gravimeter components. The overall SAE for the two new and the old FG5s are 1.14 µGal, 1.22 µGal and 1.77 µGal, respectively.

The result of the estimation 2 for the MPG-2 gravimeter is 1.3 µGal with an estimated uncertainty of 0.4 µGal. The corresponding reference height is 1.14 m. This is very similar to that of the estimation 1, i.e. 1.4 µGal ± 0.3 µGal. On average, the SAE is 1.4 µGal which will be used for the final evaluation.

Table 2.3.2 SAE of the FG5 gravimeters at the effective position of the free fall

Interferometer type	fiber laser optics (New FG5)		bulk (Old FG5)
Dropper tripod type Effective height	straight legs 1.21 m	sloping legs 1.22 m	sloping legs 1.22 m
Component	E_{SAE} /µGal	E_{SAE} /µGal	E_{SAE} /µGal
DROPPING CHAMBER			
Top flange and viewport	-0.16	-0.16	-0.16
Aluminum part of the Dropper	0.01	0.01	0.01
Steel part of Dropper + ion pump	0.52	0.52	0.52
Bottom Flange	0.27	0.27	0.27
Guide rods	0.02	0.02	0.02
Cart	-0.30	-0.30	-0.30
DROPPER TRIPOD			
Top panel	0.25	0.34	0.34
Legs + feet	0.11	0.12	0.12
SUPERSPRING	0.11	0.10	0.10
ELECTRONICS	0.03	0.03	0.03
INTERFEROMETER			
Interferometer + laser	0.23	0.22	0.78
Legs/Tripod	0.05	0.05	0.04
IN TOTAL E_{SAE}	**1.14**	**1.22**	**1.77**
Standard uncertainty u_{SAE}	**0.2**		

2.3.3 Estimation 3

Table 2.3.3 lists the SAE values reported by the owners of other absolute gravimeters.

Table 2.3.3 SAEs of the other three types of AG, as given by their owners

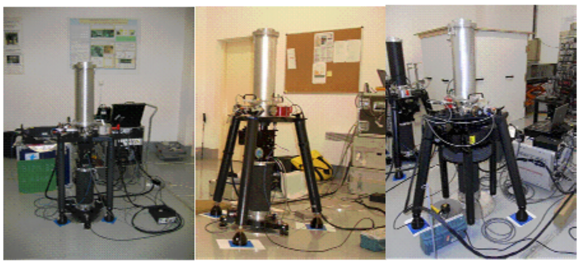
Type of AG	Ref. height/m	E_{SAE}/ µGal	u_{SAE}/ µGal	Note
NIM 2	1.44	1.17	0.03	by Wu Shuqing, 22/4/2011
IMGC 2	0.48	0.5	0.3	by Annex D, ICAG-2009
CAG	0.82	1.3	0.1	cf. [9]

2.3.4 Correction for the SAE for each AG participating in ICAG-2009

Table 2.3.4.1 lists the values of the self-attraction corrections (C_{SAC}) that will be applied to the raw AG data for ICAG-2009.

Where the estimations 1 and 2 gave different results, we used the mean value corresponding to the effective height. From table 2.3.1, the uncertainty of the estimation 1 for the type ‘Old’ of FG5 is 0.03 µGal while that of the Estimation 2 from Table 2.3.2 is 0.2 µGal. The difference of the two estimated SAEs is 0.25 µGal. A conventional uncertainty is therefore needed in the final ICAG data processing, i.e. if an uncertainty value estimated by the related evaluation is smaller than 0.2 µGal, we take the conventional uncertainty to be 0.2 µGal. Otherwise, we used the evaluated uncertainty. Figure 2.3.4 illustrates the three types of FG5 as given in the Table 2.3.4.1a.

Table 2.3.4.2 lists the AG data corrected by the SAC. Here *g* is the raw data measured at the reference height (Ref. *H*) minus 980 920 000 µGal submitted to BIPM by the participants in the Annex C and g_{SAC} is that corrected by the SAC. *u* is the associated combined standard uncertainty supplied also by the participants.



Left) FG5 Type New1 (C_{SAC} = - 1.24 ±0.2 µGal), fiber laser optic interferometer with straight legs
Middle) FG5 Type New2 (C_{SAC} = - 1.32 ±0.2 µGal), fiber laser optic interferometer with slopping legs
Right) FG5 Type Old (C_{SAC} = - 1.81 ±0.2 µGal) with bulk interferometer

Figure 2.3.4 The three types of FG5 and the corresponding *SACs*

Table 2.3.4.1 The SAC applied in the ICAG2009 evaluation
(cf. Figure 2.3.4 and Table 2.3.4.2)

Table 2.3.4.1a SAC for the FG5 gravimeter corresponding to the effective heights

AG	C_{SAC} / µGal			Height
Interferometer	fiber laser optics (New FG5)		bulk (Old FG5)	
Dropper tripod Type	straight legs New1	sloping legs New2	sloping legs Old	
Estimation 1	-	- 1.17 ± 0.2	- 1.52 ± 0.2	1.22 m[3]
Estimation 2	- 1.14 ± 0.2	- 1.22 ± 0.2	- 1.77 ± 0.2	1.22 m
Mean [4]	- 1.12 ± 0.2	- 1.20 ± 0.2	- 1.65 ± 0.2	1.22 m

Table 2.3.4.1b SAC for the gravimeters participating in ICAG-2009, corresponding to the effective height where available (FG5 models) or otherwise the user-given reference height

Type of AG	Height/m	C_{SAC}/ µGal	u_{SAC}/ µGal	Note
A-10	0.7 to 0.9	-0.58	±0.3	Estimation 1
JILAg	0.8 to 0.9	-0.69	±0.2	Estimation 1
MPG-2	1.142	-1.4	±0.2	Estimation 1
FGL	0.814	-1.1	±0.2	Estimation 1
NIM II	1.44	-1.17	±0.2	Owner
IMGC	0.48	-0.5	±0.3	Owner
CAG	0.82	-1.3	±0.1	Owner
FGC-1	0.235	+1.93	±0.2	Estimation 1, not for ICAG-2009

[3] SAC value referred to the reference height 1.22 m obtained by interpolation.
[4] According to the Table 2.3.2, the difference of the SAE between the New type FG5 straight legs and the slopping legs is 0.08 µGal.

Table 2.3.4.2 AG data before and after corrections for the SAE

(Column 8 gives the raw data at the reference height (*h*) as submitted to the BIPM by the participants; column 13 gives the gravity values after corrections for the SAE . *u* is the combined standard uncertainty. C_{SAC} is the SAC values; of which the uncertainty is smaller than 0.2 µGal will be set 0.2 µGal; Type New or Old is the type of AG; cf. Table 2.3.4.1a, New1 for straight legs and New2 for slopping legs, NA stands for Not Applicable)

N.	AG(*k*)	Inst.	ICAG	Type	Stn (*j*)	***u*** (*jk)*	***g*** (*jk*)	***h*** (*jk*)	***u***	***H***	C_{SAC}	g_{sac}
	2	3	4	5	6	7	8	9	10	11	12	13
1	NIM002	NIM	KC	NA	B2	6.6	27928.10	1.1870	6.67	1.1870	-1.17	27926.93
					B6	7.4	27915.70	1.1870			-1.17	27914.53
					B	6	27949.30	1.1870			-1.17	27948.13
2	CAG001	LNE-SYRTE	KC	NA	B1	6.1	28026.50	0.8160	6.27	0.8168	-1.30	28025.20
					B6	6.7	28027.30	0.8178			-1.30	28026.00
					B	6	28050.90	0.8165			-1.30	28049.60
3	FG5209	METAS	KC	New2	B5	2.9	27907.90	1.2983	2.90	1.2994	-1.32	27906.58
					B	2.9	27904.10	1.2980			-1.32	27902.78
					B2	2.9	27891.00	1.3020			-1.32	27889.68
4	FG5213	NMIJ /AIST	KC	New2	B5	2.5	27909.80	1.2772	2.50	1.2777	-1.32	27908.48
					B	2.5	27908.50	1.2781			-1.32	27907.18
					B1	2.5	27904.40	1.2779			-1.32	27903.08
5	FG5215	VUGTK /RIGTC	KC	Old	B6	2.4	27910.20	1.2119	2.40	1.2119	-1.81	27908.39
					B1	2.4	27923.60	1.2124			-1.81	27921.79
					B5	2.4	27928.50	1.2115			-1.81	27926.69
6	JIL006	BEV	KC	NA	B2	7.8	28027.90	0.8400	7.47	0.8400	-0.61	28027.29
					B5	7.3	28042.80	0.8400			-0.61	28042.19
					B1	7.3	28035.50	0.8400			-0.61	28034.89
7	FGL103	KRISS	KC	NA	B2	4.5	28020.00	0.8090	4.50	0.8143	-1.10	28018.90
					B1	4.5	28040.00	0.8090			-1.10	28038.90
					B6	4.5	28020.00	0.8250			-1.10	28018.90
8	FG5224	CMS/ ITRT	KC	New1	B6	2.8	27886.00	1.2822	2.83	1.2835	-1.12	27884.88
					B5	2.9	27902.40	1.2832			-1.12	27901.28
					B2	2.8	27886.30	1.2852			-1.12	27885.18
9	A10005	UME	KC	NA	B1	5.9	28008.70	0.9000	5.20	0.9000	-0.58	28008.12
					B6	4.8	27999.10	0.9000			-0.58	27998.52
					B	4.9	28012.60	0.9000			-0.58	28012.02
10	FG5105	NRC	KC	New1	B	2.7	27898.00	1.3110	2.70	1.3110	-1.12	27896.88
					B1	2.7	27898.70	1.3110			-1.12	27897.58
					B6	2.7	27883.80	1.3110			-1.12	27882.68
11	FG5221	FGI	KC	New1	B5	2.7	27935.90	1.2000	2.70	1.2000	-1.12	27934.78
					B2	2.7	27915.80	1.2000			-1.12	27914.68
					B1	2.7	27930.40	1.2000			-1.12	27929.28
12	A10014	IPGP	PS	NA	B	6.1	28027.57	0.9000	6.10	0.9000	-0.58	28026.99
					B2	6.1	28000.95	0.9000			-0.58	28000.37
					B6	6.1	28005.28	0.9000			-0.58	28004.70
13	A10020	IGC	PS	NA	B1	10	28070.97	0.7155	10.50	0.7155	-0.58	28070.39
					B6	10.9	28053.25	0.7155			-0.58	28052.67
					B2	10.6	28058.31	0.7155			-0.58	28057.73
14	FG5101	BKG	PS	New2	B	1.92	27906.58	1.2908	1.91	1.2903	-1.32	27905.26
					B1	1.92	27901.65	1.2902			-1.32	27900.33
					B5	1.9	27905.17	1.2900			-1.32	27903.85
15	FG5102	NOAA	PS	New1	B1	2.4	27896.05	1.2986	2.40	1.2984	-1.12	27894.93
					B2	2.4	27877.79	1.2976			-1.12	27876.67
					B	2.4	27895.78	1.2991			-1.12	27894.66
16	FG5228	UNIV. MONTP.	PS	New1	B2	2.23	27999.79	0.9000	2.23	0.9000	-1.12	27998.67
					B6	2.23	28000.83	0.9000			-1.12	27999.71
					B	2.23	28019.44	0.9000			-1.12	28018.32
17	FG5230	W.U. TECH.	PS	New1	B2	2.3	27995.37	0.9000	2.33	0.9000	-1.12	27994.25
					B	2.4	28013.97	0.9000			-1.12	28012.85
					B6	2.3	27994.85	0.9000			-1.12	27993.73
18	FG5233	LANT-MATE-RIET	PS	New1	B2	2.4	27893.10	1.2800	2.40	1.2803	-1.12	27891.98
					B1	2.4	27905.60	1.2800			-1.12	27904.48
					B	2.4	27907.40	1.2810			-1.12	27906.28
19	FG5238	INGV	PS	New1	B	2.7	27907.80	1.2797	2.77	1.2799	-1.12	27906.68
					B6	2.8	27894.70	1.2792			-1.12	27893.58
					B5	2.8	27912.00	1.2807			-1.12	27910.88
20	MPG002	MAX PLANCK INST.	PS	NA	B5	8.2	27964.20	1.1420	8.17	1.1423	-1.35	27962.85
					B	8.2	27960.70	1.1440			-1.35	27959.35
					B1	8.1	27946.80	1.1410			-1.35	27945.45
21	FG5220	IfE	PS	New1	B2	2.4	27915.80	1.2000	2.40	1.2000	-1.12	27914.68
					B	2.4	27933.20	1.2000			-1.12	27932.08
					B1	2.4	27927.90	1.2000			-1.12	27926.78

III. Influence of SAE on the CIPM KC ICAG 2009

The TP for ICAG-2009 was the version 4 issued in the Sept. 2009 [1]. The KC draft A Report version A1 and version A2 were issued separately on the 11th June 2010 and 18th Nov. 2010. The SAE started to draw the attention of the BIPM pilot laboratory in Jan. 2011. At the request of the BIPM and based on the earlier study [6], the first author of this report evaluated the SAE, the Estimation 1.

Given that the SAE appeared to be significantly larger than its uncertainty as specified in the TP and given the obvious impact of this on the KCRV, we decided to do three things:

1) Verify the SAE values given in the Estimation 1.
2) Investigate how to apply the self-attraction correction (SAC) and its impact on the evaluation of the KCRV and the offset of the KC ICAG 2009.
3) Whatever the output of the points 1) and 2), the result should be accepted and approved by all the KC participants so as to be able to use in the Draft A report. The Pilot laboratory should propose first several options for discussion.

With respect to (1), as described above an independent estimate of the SAE was made for the FG5 models (the Estimation 2). The third author of this report then carried out the following computation and study to answer the points 2) and 3).

3.1 Four possible options for application of SAC

There are four possible ways of dealing with the SAC in the evaluation of the KC ICAG 2009.

3.1.1. Option 1: Not apply the SAC

This option is the same as applying SAC=0, which conforms with the TP.

The advantage of this solution is that we do not need to modify the present Draft A Report version 2, dated 18 Nov. 2010 (A2 for short); it is hence the easiest solution. The disadvantages are that in the present, the uncertainty of the KCRV is in the range 1.0 µGal to 1.3 µGal which is equal or smaller than the SAC of FG5 which dominates the KCRV determination. The KCRV are greatly biased, by nearly 1.5 µGal, and hence the values and the uncertainties given for the offset are affected for at least the non-FG5 meters. Option 1 is then not self-consistent and does not give a state-of-the-art result with respect to modern absolute gravimetry. This will be argued over the coming years and the option is therefore rejected by many, if not most of the KC participants and the members of the Steering Committee, knowing that any KC participant can disapprove the Draft A Report so as to block the KC procedure.

3.1.2. Option 2: Apply the SAC to the OUTPUT uncertainty AFTER the LS adjustment

According to the TP, a participant has the right to decide either to apply a correction in his gravity measurements or add it as a bias to the total uncertainty. Numerically, the option 2 suggests keeping the values of the KCRV and offsets as presented in A2 but to enlarge their uncertainties to account for the biases due to the SAC. This also conforms with the TP.

In order to estimate the enlarged uncertainties, we evaluated the maximum influence of the SAC on the KC and the PS of the ICAG 2009. Please refer to section 3.1.4 below for the numerical influences of the SAC. The maximum influence is 1.7 µGal on the KCRV (11 AGs) and 1.9 µGal on the PSRV (21 AGs). In consequence, the offsets corresponding to the FG5 gravimeters changed by 0.3 µGal and those of the non-FG5 gravimeters changed by 1 µGal.

This suggests that we should enlarge the final uncertainty by:

- 1.7 µGal to the uncertainty of KCRV and 1.9 µGal to that of the PSRV
- 0.3 µGal to the uncertainty of FG5 AG
- 1 µGal to the uncertainty of non FG5 AG

Although mathematically it is not a very elegant method, the advantages of Option 2 are: 1) the accuracy of the SAC is not critical; 2) there are no changes in the current KCRV/PSRV nearly approved; 3) the biases in the KCRV/PSRV and therefore in the offsets are accounted by the enlarged uncertainties; 4) the most important, it is the only one acceptable by all the KC/PS participants and the pilot laboratory (BIPM).

See chapter 4 for more detailed numerical results of this option.

3.1.3. Option 3: Apply the SAC to the INPUT uncertainty BEFORE the LS adjustment

This corresponds to the case of "correction **not** applied" in the TP Annex D. Mathematically Option 3 is not rigorous, as we know that the SAC is a bias and is always of a negative sign in the case of the ICAG 2009. Therefore the SAC will not be averaged out by a LS adjustment.

To assess the maximum influence, we simply added:

- 2 µGal to uncertainties of FG5 AGs
- 1 µGal to uncertainties of non FG5 AGs

The results are shown in Tables 3.1.3a and 3.1.3b.

Table 3.1.3a. The KCVR and PSRV of solution 3

No.	Stn	KCRV/µGal	PSRV/µGal
1.	B_.090	8020.6+-1.6	8020.1+-0.7
2.	B1.090	8013.1+-1.3	8012.9+-0.8
3.	B2.090	7999.3+-1.7	7998.8+-0.8
4.	B5.090	8021.3+-1.5	8021.0+-1.0
5.	B6.090	8000.9+-1.5	8000.5+-0.9

Table 3.1.3b. The offsets of the g-KC and g-PS calculated using solution 3

	AG	Δ(g-KC)/µGal	Δ(g-PS)/µGal
1.	NIM2	-7.8+-4.3	-8.3+-4.3
2.	CAG1	0.9+-4.1	0.5+-4.2
3.	F209	-3.2+-2.7	-3.6+-2.8
4.	F213	0.6+-2.5	0.3+-2.6
5.	F215	0.7+-2.4	0.4+-2.5
6.	J506	-6.7+-4.8	-7.0+-4.9
7.	F103	2.3+-3.1	1.9+-3.2
8.	F224	5.3+-2.6	4.9+-2.8
9.	A605	4.8+-3.5	4.4+-3.5
10.	F105	-0.9+-2.6	-1.2+-2.7
11.	F221	-2.2+-2.6	-2.6+-2.7
12.	A614		-4.8+-4.1
13.	A620		-4.2+-6.7
14.	F101		-0.3+-2.2
15.	F102		6.4+-2.5
16.	F228		-0.2+-2.4
17.	F230		5.1+-2.5
18.	F233		-0.6+-2.5
19.	F238		-2.1+-2.7
20.	M782		-9.7+-5.3
21.	F220		-1.2+-2.5

There are only small changes compared to the result of the option 2: the average changes are 0.1 ± 0.35 µGal and 0.3 ± 0.14 µGal for the key comparison and pilot study reference values, respectively; the average changes are to the individual gravimeter offsets from the KCRV and PSRV are -0.1 µGal and -0.3 ± 0.1 µGal, respectively.

The advantages of option 3 are 1) the exactitude of the SAC is not critical; 2) it agrees with TP Annex D: correction not applied but added in the total uncertainty budget as a bias. However, its major disadvantage as mentioned above is that the method is not reasonable. The bigger uncertainties in the raw data have almost negligible impact on results (compared to the solution 1) and suffer from the same inconsistency, i.e. the uncertainties of the KRCV/PSRV are less than the SAC. This is not reasonable and not acceptable, for the KC and for PS.

3.1.4. Option 4: Apply the SAC to the INPUT *g*-value BEFORE the LS adjustment

This agrees with the case of "correction applied" in the TP Annex D. Option 4 is mathematically rigorous. As we seek to know the maximum influence of the SAC on the KCRV and on the Offsets, a SAC of -2 µGal (bigger than the average about -1.5 µGal to see the maximum impact) is added to the FG5 measurement raw g values keeping in mind that the FG5 represent 90% of the total weight. It must also be pointed out that the influence of the SAE in the KCRV/PSRV is not the simple difference of the mean value out of the weighted mean values, because in the LS adjustment the offsets are constrained by the condition:

$$\sum w_k \delta_k = 0 \qquad (3.1.4)$$

where the δ_k stands for the offset of the measurement of the AG(k) with respect to the KCRV and in this case the index k runs over the gravimeters taking part in the KC with weight w_k. This condition ensures that the weighted mean of the offsets in the KC is zero.

The results are presented in Table 3.1.4a and 3.1.4b.

Table 3.1.4a The KCRV and PSRV of the solution 3

No.	Stn	KCRV/µGal	PSRV/µGal
1.	B_.090	8018.1+-1.3	8017.8+-0.6
2.	B1.090	8011.6+-1.0	8011.0+-0.6
3.	B2.090	7997.5+-1.3	7996.7+-0.7
4.	B5.090	8019.6+-1.0	8018.8+-0.8
5.	B6.090	7999.3+-1.2	7998.5+-0.8

Table 3.1.4b The offsets of the KC-AG and PS-AG of the solution 3

OffSet		Δ(g-KC)/µGal	Δ(g-PS)/µGal
1.	NIM2	-10.0+-3.8	-10.6+-3.8
2.	CAG1	-0.8+-3.5	-1.4+-3.6
3.	F209	-3.2+-1.6	-3.8+-1.6
4.	F213	0.7+-1.3	0.1+-1.4
5.	F215	1.1+-1.3	0.4+-1.4
6.	J506	-8.2+-4.2	-9.0+-4.3
7.	F103	0.7+-2.5	-0.1+-2.6
8.	F224	5.6+-1.5	4.8+-1.6
9.	A605	2.8+-2.9	2.3+-2.9
10.	F105	-0.7+-1.4	-1.3+-1.5
11.	F221	-1.9+-1.4	-2.6+-1.5
12.	A614		-6.9+-3.5
13.	A620		-6.1+-6.0
14.	F101		-0.4+-1.1
15.	F102		6.3+-1.4
16.	F228		-0.3+-1.3
17.	F230		4.9+-1.3
18.	F233		-0.7+-1.4
19.	F238		-2.3+-1.6
20.	M782		-12.2+-4.8
21.	F220		-1.3+-1.4

The advantages of the option 4 are that it is scientifically rigorous, agrees with TP Annex D the case of "correction applied" and the related Report A will be self-consistent. The disadvantages are: 1) there will be big changes to the results compared to the current report A2: changes of -1.7 µGal and -1.9 µGal to the KCRV and PSRV; 2) changes of 0.3 µGal and 0.2 µGal in the offsets of KC and PS; 3) the exactitude of the SAC is critical. However, the SAE has not yet been rigorously estimated for all the 7 types of AG involved in the key comparison because the information of the structures, the material and the density of all each pieces of all the AGs is not still completed. The accuracy of the SAC is critical in this case. Can we apply a correction which is not yet exactly evaluated?

We can roughly give the approximate relations between the SAC of FG5 and its affections to KCRV and to the offset of FG5:

SAC= Δ(KCRV)+Δ(Offset_FG5)
E.g., for Δ(KCRV) = -1.7 µGal and Δ(offset_FG5)=-0.3 µGal, we have SAC=-2 µGal.

Here Δ stands for the mean difference the value in the round parentheses with and without the SAC, e.g., Δ(Offset_FG5) is that of the offsets of the FG5 AGs. This is why in the Option 2 (Section 3.1.2), we increase 1.7 µGal in the uncertainties of KCRV/PSRV; 0.3 µgal in the uncertainty of FG5 AGs and 1 µGal in the uncertainty of non-FG5 AG.

IV. Updates the CIPM KC ICAG-2009 Report A

At that time of the meeting, the Report A2 has not been approved by all the KC participants (when the first version V0 of this report was prepared on the 21/04/2011) and can still be modified. Taking into account the different arguments, the members of the ICAG Steering Committee and the KC participants converge to the Option 2 (Section 3.1.2).

Considering that:

1. The SAE produces a non negligible bias in the KCRV;
2. The SAC value for FG5 is almost an order of magnitude bigger than its uncertainty as given in the TP Annex D: 1.8 µGal compared to 0.2 µGal;
3. Not all the SACs for all the models of the KC AGs have been correctly evaluated;
4. Further investigations should examine the exactitude of the SAC and extend the estimation to all the types of AG

It is decided to NOT apply the SAC in the KC measurements but, for sake of the self-consistency in the Draft A Report for the KC comparison CCM.G-K1, to add a large enough bias to the final uncertainty to cover the bias due to the SAC. We add:

- 1.7 µGal to the final uncertainty of the KCRVs;
- 0.3 µGal to the final uncertainty of the FG5 AG offsets and
- 1 µGal to the final uncertainty of the non-FG5 AG offsets.

The enlarged uncertainties will be listed by adding a column in Tables 2 and 3 after the column *u* (statistical standard uncertainty) in the Draft report A2. See Table 4.1a and 4.1b. The error bars in the figures do not include the SAC because the latter is basically a bias.

Including also other updates, the new version is referred to as Draft **A Report of the ICAG-2009 Key Comparison CCM.G-K1**[5]**.**

Remarks:

1. For the evaluation of the SAE's influence on the ICAGs (Section 3), we have used the mean value of the estimations 1 and 2, i.e., about 2 µGal[6]. However, the approximation is good enough for the enlarged uncertainty.

2. For the French SYRTE CAG, according to the emails of Dr Franck Pereira dated 24/04/2011 at 21h56 and 26/04/2011 at 00h50, the SAE had already been taken into account in their evaluation. There is no need to add the 1 µGal bias to the enlarged uncertainty of the offset.

Table 4.1a. The KCRV in µGal

No.	Stn	KCRV	*u*	*U*	*d* .
1.	B.090	28019.8	1.3	3.0	-0.2
2.	B1.090	28013.3	1.0	2.7	-0.5
3.	B2.090	27999.2	1.3	3.0	-0.7
4.	B5.090	28021.3	1.0	2.7	-0.7
5.	B6.090	28001.0	1.2	2.9	-0.7

Table 4.2b. Offsets for the AGs in µGal

No.	AG	ICAG	OS_{KC}	*u*	*U*	$OS_{RG}-OS_{KC}$
1.	NIM 002	KC	-8.3	3.8	4.8	-0.1
2.	CAG 001	KC	0.9	3.5	3.5	-1.2
3.	FG5 209	KC	-3.5	1.6	1.9	-1.1
4.	FG5 213	KC	0.4	1.3	1.6	-0.9
5.	FG5 215	KC	0.8	1.3	1.6	-1.2
6.	JIL 006	KC	-6.5	4.2	5.2	-1.3
7.	FGL 103	KC	2.4	2.5	2.8	0.3
8.	FG5 224	KC	5.3	1.5	1.8	-1.1
9.	A10 005	KC	4.5	2.9	3.9	-1.1
10.	FG5 105	KC	-1.0	1.4	1.7	-0.7
11.	FG5 221	KC	-2.2	1.4	1.7	-0.8
12.		PS	-4.6	3.6	4.6	-1.3
13.		PS	-3.4	3.1	4.1	-1.2
14.		PS	-0.2	1.3	1.6	-0.9
15.		PS	6.6	1.6	1.9	-1.0
16.		PS	0.0	1.5	1.8	-1.7
17.		PS	5.3	1.4	1.7	-1.3
18.		PS	-0.4	1.6	1.9	-1.0
19.		PS	-1.9	1.7	2.0	-1.2
20.		PS	-9.9	4.8	5.8	-0.9
21.		PS	-1.1	1.4	1.7	-0.9

[5] The final version of the report A was approved on 10 May 2011 during the meeting of the CCM Working Group on Gravimetry held at the BIPM.
[6] Based on the version V1 of this reported dated in the beginning of May 2011

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